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Exclusive decays of $\Xi^{\diamond}_{QQ'}$ baryons in NRQCD sum rules

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Abstract

We perform a detailed study of semileptonic form-factors for the doubly heavy baryons in the framework of three-point NRQCD sum rules. The analysis of spin symmetry relations as well as numerical results on various exclusive decay modes of doubly heavy baryons are given.

1 Introduction

Besides an experimental search for an explanation of the phenomenon of electroweak symmetry breaking and new physics beyond the Standard Model, high energies and luminosities of current and future particle accelerators provide a possibility to observe rare processes with heavy quarks. An interesting topic here is a study of physics of doubly heavy baryons. An analysis of dynamics of these baryons can play a fundamental role in an extraction of primary parameters of weak quark interactions. Due to distinctions between the QCD effects inside the doubly and singly heavy hadrons, one may strictly constrain incalculable nonperturbative quantities, entering different schemes of calculations.

The real possibility of such experimental measurements was recently confirmed by CDF Collaboration by the first observation of B_c meson [1]. As predicted theoretically [2], this long-lived state of $\bar{b}$ and c quarks has the production cross sections, mass and decay rates, which are compatible with the characteristic values for the doubly heavy hadrons. Thus, the experimental search for the doubly heavy baryons can also be successful. Of course, such the search would be more strongly motivated if it would be supported by modern theoretical studies and evaluations of basic characteristics for the doubly heavy baryons.

Some steps forward this program were already done. First, the production cross sections of doubly heavy baryons in hadron collisions at high energies of colliders and in fixed target experiments were calculated in the framework of perturbative QCD for the hard processes and factorization of soft term related to the nonperturbative binding of heavy quarks [3]. Second, the lifetimes and branching fractions of some inclusive decay modes were evaluated in the Operator Product Expansion combined with the effective theory of heavy quarks, which results in series over the inverse heavy quark masses and relative velocities of heavy quarks inside the doubly heavy diquark [4, 5]. Third, the families of doubly heavy baryons, which contain a set of narrow excited levels in addition to the ground state, were described in the framework of potential models [6]. The picture of spectra, obtained in this analysis, is very similar to that of heavy quarkonia. Fourth, the QCD and NRQCD sum rules [7] were explored for the two-point baryonic currents in order to calculate the masses and couplings of doubly heavy baryons [8, 9, 10]. And fifth, there are papers, where exclusive semileptonic and some nonleptonic decay modes of doubly heavy baryons in the framework of potential models and within the Bethe-Salpeter approach were analyzed [11, 12].

In the present paper we estimate form-factors for the semileptonic decays of doubly heavy baryons together with semileptonic and some nonleptonic decay modes in the framework of three-point NRQCD sum rules for the case of spin 1/2 - spin 1/2 - baryon transitions only. The estimates of

contributions due to spin 1/2-spin 3/2-baryon transitions are also given the basis of QCD superflavor symmetry, emerging in the limit of very large heavy quark masses. In the limit of zero recoil we derive the spin symmetry relations on the form-factors, governing the semileptonic transitions of doubly heavy baryons. The use of these symmetry relations greatly simplifies further evaluation of form-factors for doubly heavy baryon transitions. A detailed analysis of baryon couplings, needed to model the phenomenological part of three-point sum rules is also provided. Our further exposition of the obtained results is organized as follows. In Section 2 we discuss our choice of interpolating currents between vacuum and corresponding doubly heavy baryon state and give numerical estimates of baryonic couplings in the framework of two-point NRQCD sum rules. Section 3 is devoted to the description of the method used to calculate the form-factors of interest. Here we present a derivation of spin symmetry relations between form-factors in the limit of maximal invariant mass of leptonic pair and give analytical expressions for corresponding double spectral densities. In section 4 we present numerical estimates of the form-factors studied together with predictions for the semileptonic and some nonleptonic decay modes. And finally, section 5 contains our conclusion.

2 Two point sum rules

In this section we describe steps, required for the evaluation of baryonic constants, as they will be needed later to model the phenomenological part of three-point sum rules. The question, which should be solved first is the choice of the corresponding interpolating currents for the baryons under consideration. So, in the next subsection we discuss their various choices and comment on the merits of the prescription, used in this paper.

2.1 Baryonic currents

As was mentioned in Introduction, in this work we consider only spin 1/2 - spin 1/2 transitions of doubly heavy baryons. Hence, the discussion of baryonic currents later in this subsection will be restricted to this case. For baryons, containing two heavy quarks, there are two types of interpolating currents:

1) The prescription with the explicit spin structure of the heavy diquark from the very beginning is given by

$$\begin{aligned} J_{\Xi'^{\diamond}_{QQ'}} &= [Q^{iT} C\tau\gamma_5 Q^{j\prime}] q^k \varepsilon_{ijk}, \\ J_{\Xi^{\diamond}_{QQ}} &= [Q^{iT} C\tau\boldsymbol{\gamma}^m Q^j] \cdot \boldsymbol{\gamma}_m \gamma_5 q^k \varepsilon_{ijk}, \end{aligned} \tag{1}$$

Here C is the charge conjugation matrix with the properties $C\gamma_\mu^T C^{-1} = -\gamma_\mu$ and $C\gamma_5^T C^{-1} = \gamma_5$, i, j, k are color indices and τ is a matrix in the flavor space.

2) The currents, which require a further symmetrization of heavy diquark wave function, have the form

$$J_{\Xi^{\diamond}_{QQ'}} = \varepsilon^{\alpha\beta\gamma} : (Q^T_\alpha C\gamma_5 q_\beta) Q'_\gamma : \tag{2}$$

The currents of the second type can be easily related to those of first type by the Fierz rearrangement of quark fields and the further symmetrization or antisymmetrization of diquark wave-function, depending on the diquark spin state. The first type of these currents was considered in [9, 10],

while the currents of second type were discussed in [8]. The evaluation of form-factors, describing semileptonic decays of doubly heavy baryons, what is the goal of this paper, is much easy if we use the currents of the second type and make the symmetrization or antisymmetrization of diquark wave-function at the end of calculation on the level of form-factors. This procedure will become clear in the section with our numerical results for the exclusive decay modes of doubly heavy baryons. The reasons here are the manifest symmetry relations for form-factors, discussed by us later and rather simple calculations of various spectral densities for three-point correlation functions, which can also be done in full QCD framework without performing complicated angular integrations. However, as was shown by the authors of [8], for the second type of currents it is difficult, in general, to achieve a stability of sum rules predictions for the both extracted mass and coupling of doubly heavy baryons. To the same time, all these difficulties are absent for the currents of first type. So, the conclusion, which one may do here, is that, the appropriate choice of baryonic currents depends on the problem, which you would like to solve.

Thus, in view of the lack of experimental information on the masses of doubly heavy baryons we will use for them the results of two-point NRQCD sum rules for the first type of currents [9, 10]. Taking these mass values as input, we calculate further the baryonic couplings of the second type.

Next, let us discuss the couplings of strange heavy baryons, as they are appear in the semileptonic decays of some doubly heavy baryons. The currents, describing these hadrons, also classified according to the symmetry properties of light diquark wave-function. There are two spin 1/2 Λ-type (antisymmetric in the q and s - quarks[1]) and two Σ-type (symmetric in the q and s - quarks) HQET currents, namely

$$\begin{aligned} J_{\Lambda 1} &= (q^T C\gamma_5 s)Q_v, \quad J_{\Lambda 2} = (q^T C\gamma^0\gamma_5 s)Q_v, \\ &\qquad (\Lambda \text{ - type}) \\ J_{\Sigma 1} &= (q^T C\gamma^k s)\gamma^k\gamma_5 Q_v, \quad J_{\Sigma 2} = (q^T C\gamma^0\gamma^k s)\gamma^k\gamma_5 Q_v, \\ &\qquad (\Sigma \text{ - type}) \end{aligned} \tag{3, 4}$$

where Q_v is the HQET heavy quark spinor, moving with velocity v. The baryons, described by these currents, belong to the same SU(4) multiplet, that have as its lowest level the $J^P = \frac{1}{2}^+$ SU(3) octet. As, these hadrons contain only one heavy quark, they belong to the second level of the mentioned SU(4) multiplet. This level splits apart into two SU(3) multiplets, a $\bar{\mathbf{3}}$, states of which Ξ_Q are antisymmetric under interchange of two light quarks and thus described by Λ - type currents, and $\mathbf{6}$, states of which Ξ'_Q are symmetric under interchange of light quark and described by Σ - type currents. Actually, there may be some mixing between the pure $\bar{\mathbf{3}}$ and $\mathbf{6}$ states to form the physical Ξ_Q and Ξ'_Q states[2]. So, in what follows we will not distinguish between Ξ_Q and Ξ'_Q - baryons and will exploit the fact that both states have non-vanishing overlap with $\varepsilon^{\alpha\beta\gamma} : (q^T_\alpha C\gamma_5 Q_\beta)s_\gamma :$ current. In other words, what we suppose to calculate is the semileptonic branching ratio of some of doubly heavy baryons into both Ξ_Q and Ξ'_Q - baryons.

Now, let us briefly describe the two-point NRQCD sum rules, used for their evaluation.

[1] Here q denotes one of the light quarks u or d.

[2] They both have the same I, J and P quantum numbers.

2.2 Description of the method

We start from the correlator of two baryonic currents with the half spin

$$\Pi(p^2) = i\int d^4x e^{ipx}\langle 0|T\{J(x),\bar{J}(0)\}|0\rangle = \not{p}F_1(p^2) + F_2(p^2), \tag{5}$$

Performing the OPE expansion of this correlation function, we get a series, different terms of which give us the contributions of operators with various dimensions. So, for F_i $(i,2)$ functions we have the following expressions

$$F_i(p^2) = F_i^{pert}(p^2) + F_i^{\bar{q}q}(p^2)\langle\bar{q}q\rangle + F_i^{G^2}(p^2)\langle\frac{\alpha_s}{\pi}G^2\rangle + F_i^{mix}(p^2)\langle\bar{q}Gq\rangle + \dots \tag{6}$$

To obtain theoretical expressions for the Wilson coefficients, standing in front of different operators, one typically uses the dispersion relation

$$F_i^{\diamond}(t) = \frac{1}{\pi}\int_0^{\infty}\frac{\rho_i^{\diamond}(w)dw}{w-t}, \tag{7}$$

where $t = k\cdot v$, $p_\mu = k_\mu + (m_1+m_2)v_\mu$ and $\rho_i^{\diamond}$ denotes the imaginary part in the physical region of corresponding Wilson coefficients in NRQCD[3]. The calculation of spectral densities $\rho_i^{\diamond}$ proceeds through the use of Cutkosky rules [14] and for the case of $\varepsilon^{\alpha\beta\gamma}:(Q_\alpha^T C\gamma_5 q_\beta)Q'_\gamma:$ current and different quark masses was done in the QCD framework in [8]. Here we use the results of this work. The needed NRQCD spectral densities were obtained by simple NRQCD expansion of corresponding QCD expressions and the results of this expansion could be found in Appendix A.

To relate the NRQCD correlators to hadrons, we use the dispersion representation for the two-point function with the physical spectral density, given by appropriate resonance and continuum part. The coupling constants of doubly heavy baryons are defined by the following expression

$$\langle 0|J_H|H(p)\rangle = iZ_H u(v,M_H)e^{ip\cdot x}, \tag{8}$$

where $p = M_H v$ and the spinor field with four-velocity v and mass M_H satisfies the equation $\not{v}u(v,M_H) = u(v,M_H)$.

We suppose that the continuum part, starting from the threshold value w_{cont}, is equal to that of calculated in the framework of NRQCD. Then, equalizing the correlators, calculated in NRQCD and given by the physical states, the integrations above w_{cont} cancel each other in both sides of sum rules relation. Further, we write down the correlators at the deep under-threshold point $t_0 = -(m_1+m_2)+t$ at $t\to 0$.

Introducing the following notation for the n-th moment of two-point correlation function

$$\mathcal{M}_n^i = \frac{1}{\pi}\int_0^{w_{cont}}\frac{\rho_i(w)dw}{(w+m_1+m_2)^n}, \tag{9}$$

and using the approximation of single bound state pole, we can write the following relation

$$\mathcal{M}_n^i = |Z_H^{[i]}|^2\frac{1}{M_H^{n+1-i}}. \tag{10}$$

[3]Here m_1 and m_2 are the heavy quark masses and v_μ is four-velocity of the baryon under consideration.

From which one can read off the corresponding expression for the baryon coupling in the moment scheme

$$|Z_H^{[i]}|^2 = \mathcal{M}_n^i M_H^{n+1-i}, \tag{11}$$

where we see the dependence of sum rules on the scheme parameter n. Therefore, we will tend to find the region of parameter values, where, first, the result is stable under the variation of moment number n, and, second, the both correlators F_1 and F_2 reproduce equal values of coupling constants. In the QCD sum rule analysis of [8] there was a problem, that values of baryon coupling constants, obtained from F_1 and F_2 correlation functions significantly differ. To cure this problem, in [9] it was proposed to include in calculations also contributions, coming from the OPE expansion for the correlator of two quark fields [13]

$$\langle 0|T\{q_i^a(x)\bar{q}_j^b(0)\}|0\rangle = -\frac{1}{12}\delta^{ab}\delta_{ij}\langle\bar{q}q\rangle \cdot \left[1 + \frac{m_0^2x^2}{16} + \frac{\pi^2x^4}{288}\langle\frac{\alpha_s}{\pi}G^2\rangle + ...\right]. \tag{12}$$

With an account of these corrections the quark condensate contribution to moments gets modified

$$\mathcal{M}_n^{\bar{q}q} = \mathcal{M}_n^{\bar{q}q} - \frac{(n+2)!}{n!}\frac{m_0^2}{16}\mathcal{M}_{n+2}^{\bar{q}q} + \frac{(n+4)!}{n!}\frac{\pi^2}{288}\langle\frac{\alpha_s}{\pi}G^2\rangle\mathcal{M}_{n+4}^{\bar{q}q}. \tag{13}$$

The derivation of two-point HQET sum rules for the heavy baryons with the strangeness follows the same lines as that for baryons with two heavy quarks. Here we only comment on the differences. To obtain the HQET expressions for the spectral densities we again use the QCD result of [8]. However, the transition between the QCD expressions for the doubly heavy baryons and HQET expressions for the heavy baryons with the strangeness is more intricate. First, we should take a limit when one of the heavy quark masses goes to zero and second, we should allow this quark to condense. We have calculated explicitly the s -quark condensate contribution to the both F_1 and F_2 correlation functions and subtracted $1/m_s$ poles from gluon condensate contribution, related to the strange quark condensate due to the following heavy quark expansion

$$\langle\bar{s}s\rangle \Rightarrow -\frac{1}{12m_s}\frac{\alpha_s}{\pi}\langle G^2\rangle - \frac{1}{360m_s^3}\frac{\alpha_s}{\pi}\langle G^3\rangle + \dots \tag{14}$$

The appearing logarithmic singularities can be related to the mixed quark condensate with the help of the same heavy quark expansion

$$\langle\bar{s}Gs\rangle \Rightarrow \frac{m_s}{2}\log m_s^2\frac{\alpha_s}{\pi}\langle G^2\rangle - \frac{1}{12m_s}\frac{\alpha_s}{\pi}\langle G^3\rangle + \dots \tag{15}$$

And, finally, one must subtract the nonsingular gluon condensate contribution, belonging to the quark condensate, what can be easily done with the calculated explicit expression for quark condensate contribution. The resulted HQET spectral densities for the case of ordinary baryons with strangeness were collected by us in Appendix A. For the strange quark condensate contribution, as in the case of light quark condensate, we also take into account the corrections, coming from the OPE expansion for the correlator of two strange quark fields [10]

$$\begin{aligned}\langle 0|T\{s_i^a(x)\bar{s}_j^b(0)\}\rangle &= -\frac{1}{12}\delta^{ab}\delta_{ij}\langle\bar{s}s\rangle \cdot \left[1 + \frac{x^2(m_0^2 - 2m_s^2)}{16} + \frac{x^4(\pi^2\langle\frac{\alpha_s G^2}{\pi}\rangle - \frac{3}{2}m_s^2(m_0^2 - m_s^2))}{288}\right] \\ &+ im_s\delta^{ab}x_\mu\gamma_{ij}^\mu\langle\bar{s}s\rangle\left[\frac{1}{48} + \frac{x^2}{24^2}\left(\frac{3m_0^2}{4} - m_s^2\right)\right]. \end{aligned}\tag{16}$$

With this corrections the s-quark contribution to the moments for the F_1 and F_2 correlation functions has the form

$$\mathcal{M}_1^{\bar{s}s}(n) = -\frac{1}{4} m_s \frac{(n+1)!}{n!} \mathcal{M}^{\bar{s}s}(n+1) + \frac{1}{48}\left(\frac{3m_0^2}{4} - m_s^2\right) m_s \frac{(n+3)!}{n!} \mathcal{M}^{\bar{s}s}(n+3), \tag{17}$$

$$\mathcal{M}_2^{\bar{s}s}(n) = \mathcal{M}^{\bar{s}s}(n) - \frac{m_0^2 - 2m_s^2}{16} \frac{(n+2)!}{n!} \mathcal{M}^{\bar{s}s}(n+2) + \frac{(\pi^2 \langle\frac{\alpha_s G^2}{\pi}\rangle - \frac{3}{2} m_s^2 (m_0^2 - m_s^2))}{288} \frac{(n+4)!}{n!} \mathcal{M}^{\bar{s}s}(n+4), \tag{18}$$

where

$$\mathcal{M}^{\bar{s}s}(n) = \frac{1}{\pi} \int_0^{w_{cont}} \frac{\rho^{\bar{s}s}(w) dw}{(w + m_Q + m_s)^n}, \tag{19}$$

and

$$\rho^{\bar{s}s}(w) = -\frac{\langle \bar{s}s \rangle (m_s + w)^2 (2m_Q + m_s + w)^2}{4\pi (m_Q + m_s + w)}. \tag{20}$$

Now, having all theoretical expressions for baryon coupling constants in the moment scheme, we will proceed in the next subsection with the numerical estimates.

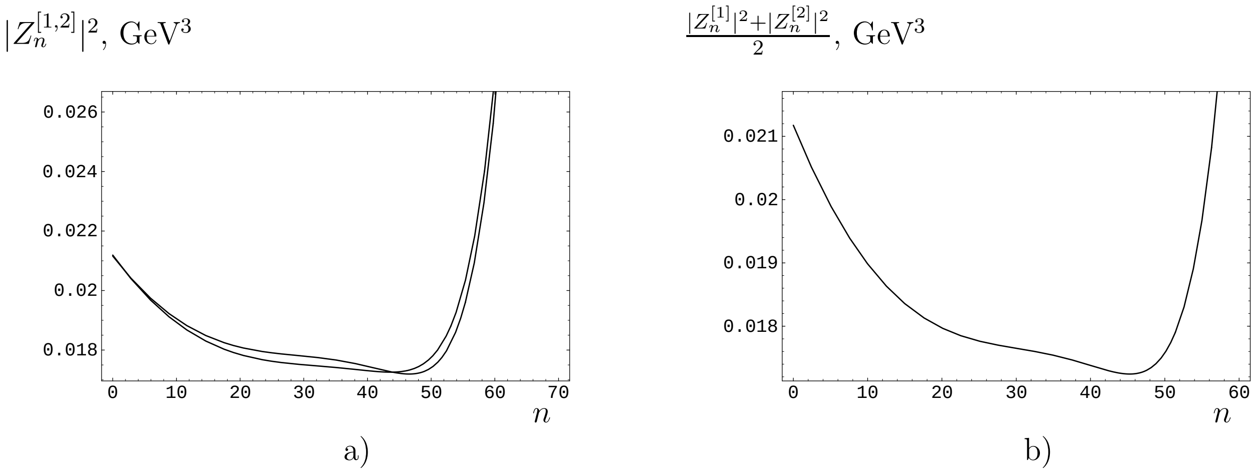


Figure 1: a) The values of $|Z_n^{[1]}|^2$ and $|Z_n^{[1]}|^2$ $\Xi_{bb}^{\diamond}$ - baryon couplings as functions of the moment number n; b) the value of $\frac{|Z_n^{[1]}|^2+|Z_n^{[2]}|^2}{2}$ average as function of the moment number n for the case of $\Xi_{bb}^{\diamond}$ - baryons.

2.3 Numerical estimates

In this subsection we present the results on the coupling constants of doubly heavy baryons and the heavy baryons with the strangeness. In the scheme of moments, which we employ here to extract the baryonic couplings from the two-point sum rules, the dominant uncertainty in estimates comes from the variation of heavy quark mass values. In the analysis we chose the following region of quark mass values

$$m_b = 4.6 - 4.7 \text{ GeV}, \quad m_c = 1.35 - 1.40 \text{ GeV}, \tag{21}$$

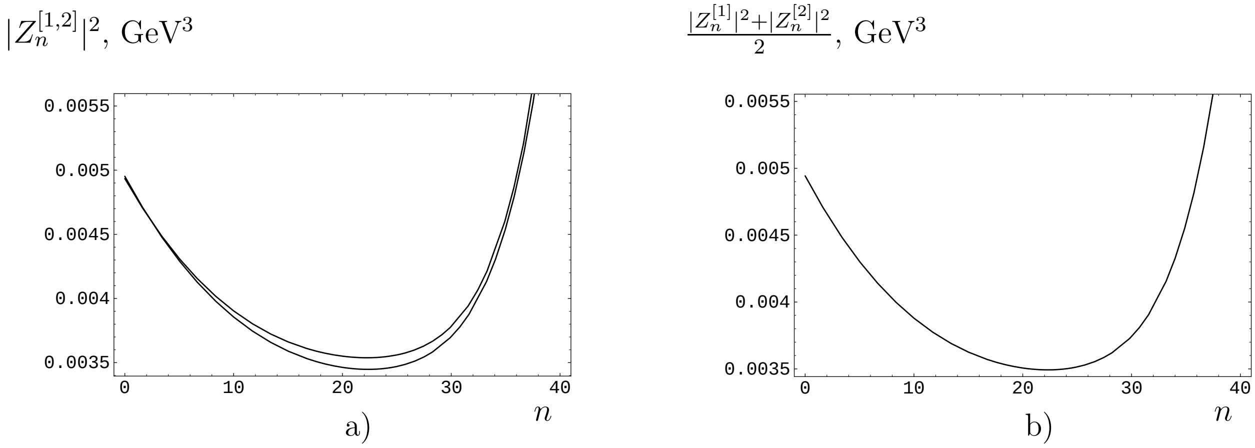


Figure 2: The results for the $\Xi_{bc}^{\diamond}$ - baryon interpolating current in the case of decaying b-quark (the $(c^T C\gamma_5 q)b$-current): a) the values of $|Z_n^{[1]}|^2$ and $|Z_n^{[1]}|^2$ $\Xi_{bc}^{\diamond}$ - baryon couplings as functions of the moment number n; b) the value of $\frac{|Z_n^{[1]}|^2+|Z_n^{[2]}|^2}{2}$ average as function of the moment number n for the case of $\Xi_{bc}^{\diamond}$ - baryons.

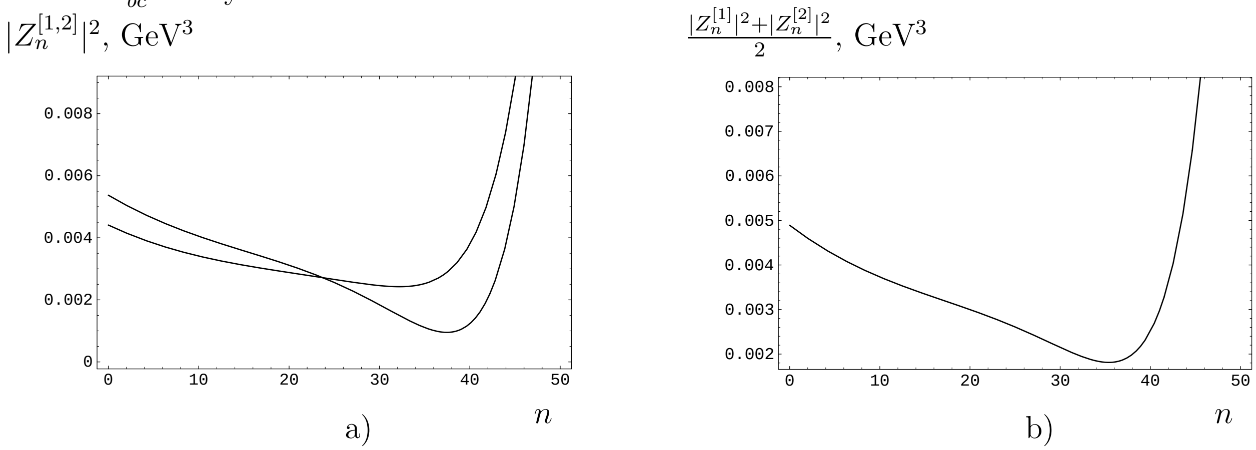


Figure 3: The results for the $\Xi_{bc}^{\diamond}$ - baryon interpolating current in the case of decaying c-quark (the $(b^T C\gamma_5 q)c$-current): a) the values of $|Z_n^{[1]}|^2$ and $|Z_n^{[1]}|^2$ $\Xi_{bc}^{\diamond}$ - baryon couplings as functions of the moment number n; b) the value of $\frac{|Z_n^{[1]}|^2+|Z_n^{[2]}|^2}{2}$ average as function of the moment number n for the case of $\Xi_{bc}^{\diamond}$ - baryons.

what is the ordinary choice used in sum rules estimates of heavy quarkonia. For the strange quark mass we use the value $m_s = 0.15$ GeV.

Next point, we would like to discuss, is an account of Coulomb corrections inside the doubly heavy diquark. As is well known, these corrections give large contribution to baryon coupling constants and are essential for relative contributions of perturbative and condensate terms to the correlator [15]. With a good accuracy at low or moderate values of moment number, the effect of Coulomb interactions can be written as overall Zommerfeld factor in front of perturbative spectral density of heavy subsystem for the square of baryon coupling. But, as it was shown in [15], the same Zommerfeld factors should be taken into account in calculations of three-point correlation functions considered later in this paper. It occurs, that for the form-factors for the semileptonic transitions of doubly heavy baryons these corrections cancel each other in average. So, the calculation of desired form-factors for the doubly heavy baryons can be consistently performed without accounting for Coulomb corrections either, provided we neglect them both in the two-point and three-point sum

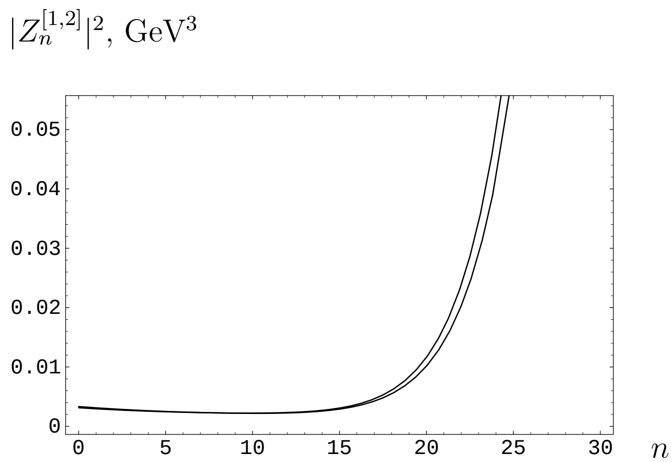


Figure 4: The values of $|Z_n^{[1]}|^2$ and $|Z_n^{[2]}|^2$ $\Xi_{cc}^{\diamond}$ - baryon couplings as functions of the moment number n.

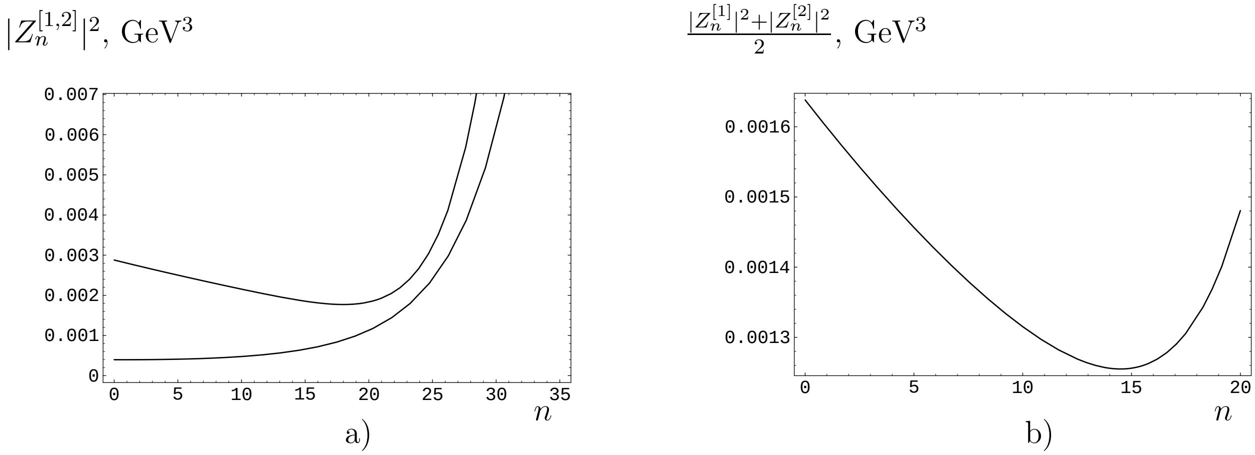


Figure 5: a) The values of $|Z_n^{[1]}|^2$ and $|Z_n^{[2]}|^2$ $\Xi_b^{\diamond}$ - baryon couplings as functions of the moment number n; b) the value of $\frac{|Z_n^{[1]}|^2+|Z_n^{[2]}|^2}{2}$ average as function of the moment number n for the case of $\Xi_b^{\diamond}$ - baryons.

rules. This is the approach, we will follow in the present work for the evaluation of form-factors.

The dependence of estimates on the threshold of continuum contribution in the two-point sum rules is not so valuable as on quark masses. We fix the region of w_{cont} as

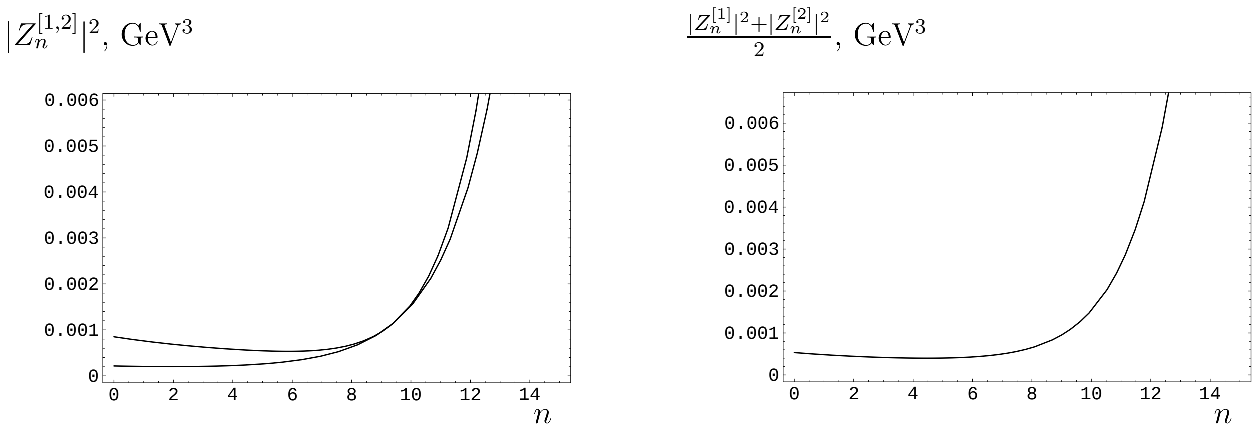
$$w_{cont} = 1.3 - 1.4 \text{ GeV}, \tag{22}$$

which is in agreement with our previous estimates of doubly heavy baryon coupling of the first type currents in the same framework of two-point NRQCD sum rules. For the condensates of quark and gluons we use the following regions:

$$\langle \bar{q}q \rangle = -(250 - 270 \text{ MeV})^3, \quad m_0^2 = 0.75 - 0.85 \text{ GeV}^2, \quad \langle \frac{\alpha_s}{\pi} G^2 \rangle = (1.5 - 2) \cdot 10^{-2} \text{ GeV}^4, \tag{23}$$

and

$$\langle \bar{s}s \rangle = 0.8 \pm 0.2 \ \langle \bar{q}q \rangle \tag{24}$$



a) b)

Figure 6: a) The values of $|Z_n^{[1]}|^2$ and $|Z_n^{[2]}|^2$ $\Xi_c^\diamond$ - baryon couplings as functions of the moment number n; b) the value of $\frac{|Z_n^{[1]}|^2+|Z_n^{[2]}|^2}{2}$ average as function of the moment number n for the case of $\Xi_c^\diamond$ - baryons.

As we already mentioned, for the second type of currents used here, we evaluate the coupling constants only and use the masses of doubly heavy baryons, calculated by us previously [9], as inputs

$$M_{\Xi_{cc}} = 3.47 \pm 0.05 \text{ GeV}, \quad M_{\Xi_{bc}} = 6.80 \pm 0.05 \text{ GeV}, \quad M_{\Xi_{bb}} = 10.07 \pm 0.09 \text{ GeV}, \tag{25}$$

which are in agreement with the values obtained in the framework of potential models. For the masses of heavy baryons with the strangeness, appearing as products of semileptonic decays of some of the doubly heavy baryons, we use the following values:

$$M_{\Xi_b} = 5.8 \text{ GeV}, \quad M_{\Xi_c} = 2.45 \text{ GeV}. \tag{26}$$

Figs. 1-6 show the dependence of baryon couplings on the momentum number. We find that the stability regions for $|Z_{1(2)}|^2$ determined from the F_1 and F_2 correlators coincide with those, obtained in the analysis of two-point correlation functions for the first type of currents [9]. However, for some of the couplings, calculated here, we see a sizeable difference in the predictions coming from the F_1 and F_2 correlation functions. This problem could not be alleviated by the variation of parameters. So, in order to determine the corresponding coupling values to be used in the phenomenological part of three-point sum rules, we consider an average coupling for these currents, whose square is given by the average of squares for Z_1 and Z_2 couplings. The resulted values of baryonic coupling are

$$|Z_{bb}|^2 = 1.7\ 10^{-2} \text{ GeV}^6, \quad |Z_{cc}|^2 = 2.3\ 10^{-3} \text{ GeV}^6, \tag{27}$$

$$\begin{aligned} |Z_{bc}^1|^2 &= 3.5\ 10^{-3} \text{ GeV}^6, & |Z_{bc}^2|^2 &= 1.8\ 10^{-3} \text{ GeV}^6, \\ (c^T C\gamma_5 q)b &- \text{current} & (b^T C\gamma_5 q)c &- \text{current} \\ |Z_{bs}|^2 &= 1.3\ 10^{-3} \text{ GeV}^6, & |Z_{cs}|^2 &= 4.3\ 10^{-4} \text{ GeV}^6. \end{aligned} \tag{28}$$

Having estimates for the couplings of initial and final state baryons with respect to semileptonic transitions, we will continue in the next section with the determination of form-factors.

3 Three-point sum rules

In this section we describe our framework for the calculation of form-factors, governing the semileptonic decays of doubly heavy baryons. Here we derive the spin symmetry relations between various form-factors, arising in the limit of the maximal invariant mass of leptonic pair and give analytical expressions for corresponding spectral densities.

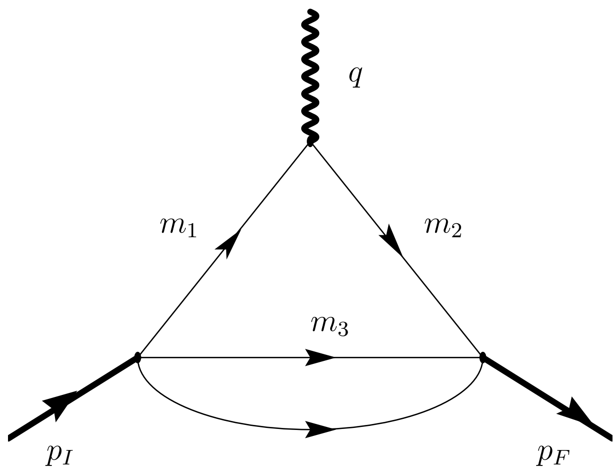


Figure 7: The diagram, corresponding to the three-point correlation function considered in the paper.

Following the standard procedure for the evaluation of form-factors in the framework of QCD sum rules, we consider the three-point function

$$\Pi_\mu = i^2 \int d^4x d^4y \langle 0|T\{J_{H_F}(x) J_\mu(0) \bar{J}_{H_I}\}|0\rangle e^{ip_F\cdot x} e^{-ip_I\cdot y}, \tag{29}$$

where J_μ is the vector or axial transition current, matrix elements of which between baryonic ground states we would like to calculate. Fig.7 shows a diagram, corresponding to the mentioned three-point function. The theoretical expression for the three-point correlation function can be easily calculated with the use of double dispertion relation

$$\Pi_\mu^{(theor)}(s_1, s_2, q^2) = \frac{1}{(2\pi)^2}\int_{m_I^2}^{\infty} ds_1 \int_{m_F^2}^{\infty} ds_2 \frac{\rho_\mu(s_1, s_2, q^2)}{(s_1 - s_1^0)(s_2 - s_2^0)} + \text{subtractions}, \tag{30}$$

where the desired spectral density could be obtained with the help of Cutkosky rules [14]. We will continue with the calculation of spectral densities later in this paper after discussing the spin symmetry relations for form-factors. The latter, as will be seen, greatly simplify the calculations to be done. Now, let us discuss the phenomenological part of three point sum rules under consideration. Saturating the channels of initial and final state hadrons by ground states of corresponding baryons, we have the following phenomenological expression for the three-point correlation function:

$$\begin{aligned}\Pi_\mu^{(phen)}(s_1, s_2, q^2) \quad &= \quad \sum_{spins} \frac{\langle 0|J_{H_F}|H_F(p_F)\rangle}{s_2^0 - M_{H_F}^2} \times \\ &\quad \langle H_F(p_F)|J_\mu|H_I(p_I)\rangle \frac{\langle H_I(p_I)|\bar{J}_{H_I}|0\rangle}{s_1^0 - M_{H_I}^2}\end{aligned} \tag{31}$$

The formfactors for the weak spin $\frac{1}{2}$ – spin $\frac{1}{2}$ baryon transitions are usually modeled as following:

$$\begin{aligned} \langle H_F(p_F)|J_\mu|H_I(p_I)\rangle &= \\ &\overline{u}(p_F)[\gamma_\mu(F_1^V + F_1^A\gamma_5) + i\sigma_{\mu\nu}q^\nu(F_2^V + F_2^A\gamma_5) + q_\mu(F_3^V + F_3^A\gamma_5)]u(p_I) \end{aligned} \tag{32}$$

However, in the NRQCD limit it is more convenient to use an alternative parametrization

$$\begin{aligned} \langle H_F(p_F)|J_\mu|H_I(p_I)\rangle &= \\ &\bar{u}(p_F)\left(\gamma_\mu G_1^V + v_\mu^I G_2^V + v_\mu^F G_3^V + \gamma_5(\gamma_\mu G_1^A + v_\mu^I G_2^A + v_\mu^F G_3^A)\right)u(p_I), \end{aligned} \tag{33}$$

where these two parametrizations can be related to each other with the help of the following relations:

$$\begin{aligned} F_1^V(t) &= G_1^V + (m_F + m_I)\left(\frac{1}{2m_I}G_2^V + \frac{1}{2m_F}G_3^V\right), \\ F_2^V(t) &= -\frac{1}{2m_I}G_2^V - \frac{1}{2m_F}G_3^V, \\ F_3^V(t) &= -\frac{1}{2m_I}G_2^V + \frac{1}{2m_F}G_3^V, \\ F_1^A(t) &= -G_1^A - (m_F - m_I)\left(\frac{1}{2m_I}G_2^A + \frac{1}{2m_F}G_3^A\right), \\ F_2^A(t) &= \frac{1}{2m_I}G_2^A + \frac{1}{2m_F}G_3^A, \\ F_3^A(t) &= \frac{1}{2m_I}G_2^A - \frac{1}{2m_F}G_3^A. \end{aligned} \tag{34}$$

Naively, all these six formfactors in either parametrization are independent, but, as we will show in the next subsection, in the limit of zero recoil the semileptonic decays of doubly heavy baryons can be described by the only universal function, an analogue of Isgur-Wise function.

3.1 Symmetry relations

Now, let us discuss the spin symmetry relations among the form-factors, arising in the limit of zero recoil for the final state baryon. That is, we consider a limit[4], where $v_I \neq v_F$ and $\omega = (v_I \cdot v_F) \to 1$. The theoretical expression for the three-point correlation function for the case of heavy to heavy underlying quark transition in this limit has the following form:

$$\Pi_\mu^{(theor)} \sim \xi^{IW}(q^2)(1 + \tilde{\not{v}}_F)\gamma_\mu(1-\gamma_5)(1 + \tilde{\not{v}}_I), \tag{35}$$

where

$$\tilde{v}_I = v_I + \frac{m_3}{2m_1}(v_I - v_F) \tag{36}$$

$$\tilde{v}_F = v_F + \frac{m_3}{2m_2}(v_F - v_I). \tag{37}$$

[4]For the discussion of this limit see [15].

So, for this type of transitions we have already, from the very beginning, the only universal function and no further analysis is required. The theoretical expression for the three-point correlation function in the case of heavy to light underlying quark transition has more complicated form

$$\Pi_\mu^{(theor)} \sim \{\xi_1(q^2)\not{v}_I + \xi_2(q^2)\not{v}_F + \xi_3(q^2)\}\gamma_\mu(1-\gamma_5)(1+\not{v}_I) \tag{38}$$

Considering different convolutions of the theoretical and phenomenological three-point correlation functions with Lorenz structures made of hadron velocities and γ - matrices and equating them, we obtain two relations on the semileptonic form-factors in this case

$$\begin{aligned} (G_1^V + G_2^V + G_3^V) &= \xi^{IW}(q^2) && (39)\\ G_1^A &= \xi^{IW}(q^2) && (40) \end{aligned}$$

and a relation between $\xi_i(q^2)$ functions

$$\xi_1(q^2) + \xi_2(q^2) = \xi_3(q^2) = \xi^{IW}(q^2)$$

Recalling also, that in any considered transition we always have heavy baryons in initial and final states and requiring that, the appropriate projections do not change the theoretical expression for the three-point correlation function, we may conclude, that in this limit there are only two form-factors of order unity $G_1^V = G_1^A = \xi(w)$, while all others are suppressed by heavy quark masses.

Having derived the spin symmetry relations, we came to situation where we should calculate the only universal function in order to obtain estimates on semileptonic or nonleptonic transitions of doubly heavy baryons.

3.2 Spectral densities

As, we said before, the calculation of spectral densities is straightforward with the use of Cutkosky rules for the quark propagators. However, the resulted expressions for NRQCD spectral densities are different[5] for the cases of heavy to heavy or heavy to light underlying quark transitions, so below we have classified the calculated quantities. For the trace of correlation function with v_μ^I we have

1) heavy to heavy underlying transition

$$\begin{aligned} \rho^{pert} &= \frac{3m_1m_2}{\sqrt{\lambda(s_I,s_F,q^2)}}(m_1^4 - 4m_1^2m_3^2 + 3m_3^4 - 4m_1^3\sqrt{s_I} + 8m_1m_3^2\sqrt{s_I} + \\ &\quad 6m_1^2 s_I - 4m_3^2 s_I - 4m_1 s_I^{3/2} + s_I^2 + 4m_3^4 \log\frac{\sqrt{s_I}-m_1}{m_3}), && (41)\\ \rho^{\bar{q}q} &= -\frac{4m_1m_2m_3\sqrt{s_I}}{(m_1+m_3)\sqrt{\lambda(s_I,s_F,q^2)}}\langle\bar{q}q\rangle, && (42) \end{aligned}$$

2) heavy to light underlying transition

[5] It is simply an artifact of NRQCD approximation.

$$\begin{aligned}\rho^{pert} &= \frac{1}{4(2\pi)^2}\frac{m_1}{\sqrt{s_I\lambda(s_I,s_F,q^2)}}[-2(m_1-\sqrt{s_I})^6+3(m_1-\sqrt{s_I})^4(m_1^2+2m_3^2- \\ &q^2+2m_2\sqrt{s_I}+s_F)-6m_3^2(m_1-\sqrt{s_I})^2(2m_1^2+m_3^2-2q^2+4m_2\sqrt{s_I}+ \\ &2s_F)+m_3^4(9m_1^2+2m_3^2-9q^2+18m_2\sqrt{s_I}+9s_F)+12m_3^4(m_1^2-q^2+ \\ &2m_2\sqrt{s_I}+s_F)\log\frac{\sqrt{s_I}-m_1}{m_3}], \qquad (43)\\ \rho^{\bar{q}q} &= -\frac{m_1m_3}{(m_1+m_3)\sqrt{\lambda(s_I,s_F,q^2)}}(m_1^2-q^2+2m_2\sqrt{s_I}+s_F-m_3^2)\langle\bar{q}q\rangle, \qquad (44)\end{aligned}$$

where

$$\lambda(x,y,z)=x^2+y^2+z^2-2xy-2xz-2yz. \qquad (45)$$

and the integration region in the double dispertion relation is determined by the condition

$$-1<\frac{1}{\sqrt{\lambda(s_I,s_F,q^2)\lambda(s_I,m_1^2,m_3^2)}}((s_I+s_F-q^2)(s_I+m_3^2-m_1^2)-2s_I(s_F-m_2^2+m_3^2))<1.$$

The notations in the above expressions should be clear from Fig. 7. Having derived theoretical expressions for the three-point correlation function, we may proceed now with the evaluation of form-factors. In numerical estimates we will use the Borel scheme for the form-factor extraction and so, below we give an expression determining the universal Isgur-Wise function for the semileptonic decays of doubly heavy baryons

$$\begin{aligned}\xi^{IW}(q^2) &= \frac{1}{(2\pi)^2}\frac{1}{8M_IM_FZ_IZ_F}\int_{(m_1+m_3)^2}^{s_I^{th}}\int_{(m_1+m_2)^2}^{s_F^{th}}\rho(s_I,s_F,q^2)ds_Ids_F\times \\ &\exp(-\frac{s_I-M_I^2}{B_I^2})\exp(-\frac{s_F-M_F^2}{B_F^2}), \qquad (46)\end{aligned}$$

where B_I and B_F are the Borel parameters in the initial and final state channels.

4 Numerical results

In this section we give the results of numerical estimates on the form-factors for the spin 1/2 - spin 1/2 doubly heavy baryon transitions. Assuming the pole resonance model for the dependence of mentioned form-factors on the square of lepton pair momentum we make predictions on the semileptonic, pion and ρ - meson decay modes.

4.1 Form-factors

The analysis of NRQCD sum rules in the Borel scheme gives us the estimates of the value of Isgur-Wise (IW) function at zero recoil for different types of transitions between doubly heavy baryons, shown in Table 1.

In Figs. 9-11 we have plotted the dependence of the normalization of IW-function on the Borel parameters in the channels of initial and final state baryons. Exploring the stability of NRQCD

sum rules upon variation of these parameters just give us the results quoted above. The subtle point in the presented analysis is the choice of the threshold values in the baryon channels. In the present analysis we put the same values as in the analysis of two-point sum rules. The results on the formfactors and later their comparison with the results of potential models convince us, that we made a right choice. However, the latter are in general different from the ones used in two-point sum rule analysis in moment scheme. For the situation, where it is the case we refer the reader to [15].

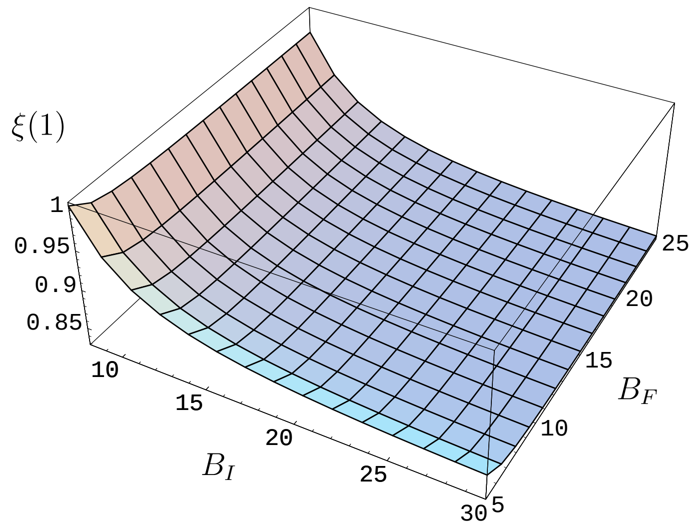


Figure 8: The value of $\xi(1)$ for the transition $\Xi_{bb}^{\diamond} \to \Xi_{bc}^{\diamond}$ as function of Borel parameters in the s_I and s_F channels.

For the sake of comparison, we also provide here the estimates of the values of IW-function at zero recoil performed by us in the framework of potential models (PM). In this approach the normalization of IW-function is given by the overlap of initial and final state baryon wave-functions. For simplicity we assume the factorization of doubly heavy baryon wave-function in the diquark and light quark - diquark wave-functions. From the HQET-description of heavy mesons we know, that the light quark affects the normalization of IW-function at zero recoil only in $1/m_Q^2$ order. Thus, in our case its affect can be neglected and the normalization of IW-function is given by the overlap of diquark wave-functions. Taking the Gaussian anzaz for the diquark wave-function we get

$$\xi^{IW}(1) = \left(\frac{2w_x w_y}{w_x^2 + w_y^2}\right)^{3/2}, \tag{47}$$

where

$$w_x = 2\pi\left(\frac{|Z_I|^2}{12}\right)^{1/3}, \quad w_y = 2\pi\left(\frac{|Z_F|^2}{12}\right)^{1/3} \tag{48}$$

and

$$|Z^{PM}| = 2\sqrt{3}|\Psi_d(0)\Psi_l(0)|. \tag{49}$$

Here we have related the parameters of diquark wavefunctions to the baryon couplings, obtained previously by us in the framework of two-point NRQCD sum rules. In table 1 we have gathered the

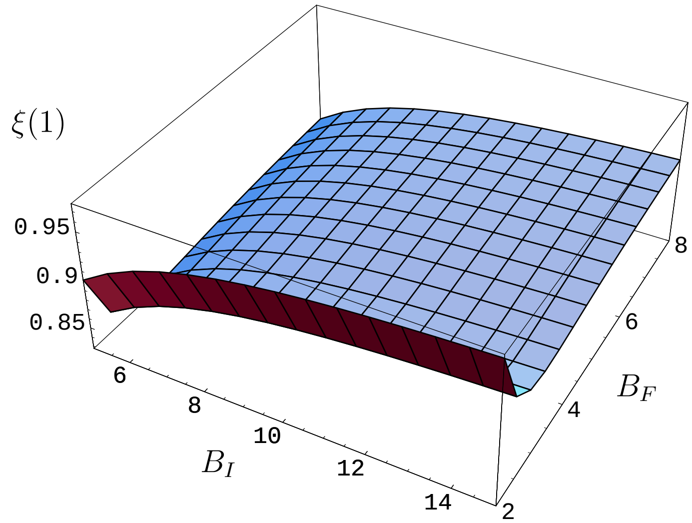


Figure 9: The value of $\xi(1)$ for the transition $\Xi_{bc}^{\diamond} \to \Xi_{cc}^{\diamond}$ as function of Borel parameters in the s_I and s_F channels.

results of sum rules and PM on the values of IW-functions at zero recoil. We see, that within the errors of sum rules method (15 %) the obtained results are very close to those of PM.

Mode	$\xi(1)$ SR	$\xi(1)$ PM
$\Xi_{bb} \to \Xi_{bc}$	0.85	0.91
$\Xi_{bc} \to \Xi_{cc}$	0.91	0.99
$\Xi_{bc} \to \Xi_{bs}$	0.9	0.99
$\Xi_{cc} \to \Xi_{cs}$	0.99	1.

Table 1: The normalization of Isgur-Wise function for different baryon transitions at zero recoil.

Next, to obtain the dependence of formfactors on the square of momentum transfer we exploit the pole resonance model. So, for the IW-function we have the following expression:

$$\xi^{IW}(q^2) = \xi_0 \frac{1}{1 - \frac{q^2}{m_{pole}^2}}, \tag{50}$$

with

$$\begin{aligned} m_{pole} &= 6.3 \quad \text{GeV for the} \quad b \to c \quad \text{transitions} \\ m_{pole} &= 1.85 \quad \text{GeV for the} \quad c \to s \quad \text{transitions.} \end{aligned}$$

4.2 Semileptonic decays

Now knowing all formfactors, describing semileptonic transitions of doubly heavy baryons we can estimate the semileptonic decay ratios for the transitions under consideration

$$Br_{SL}(\Xi_{QQ'}^{\diamond} \to \Xi_{QQ'}^{\diamond\prime}) = \tau_{\Xi_{QQ'}^{\diamond}} \int_1^{w_{max}} dw \frac{d\Gamma}{dw}(\Xi_{QQ'}^{\diamond} \to \Xi_{QQ'}^{\diamond\prime}), \tag{51}$$

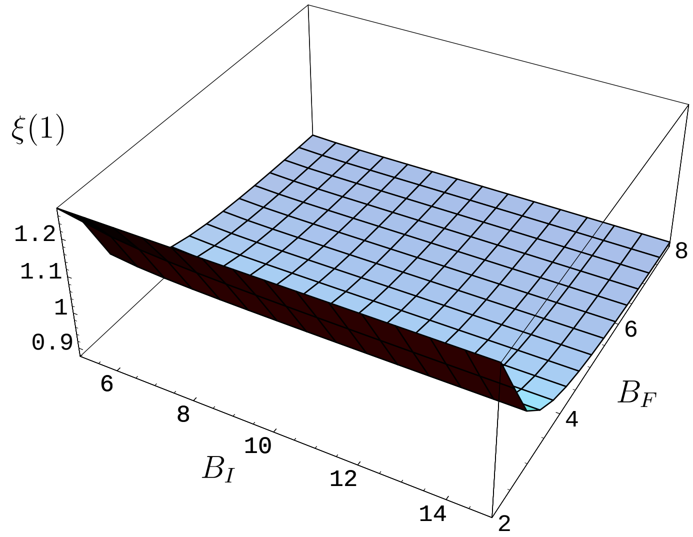


Figure 10: The value of $\xi(1)$ for the transition $\Xi_{bc}^{\diamond} \to \Xi_{b}^{\diamond}$ as function of Borel parameters in the s_I and s_F channels.

where

$$w_{max} = \frac{M_I^2 + M_F^2 - m_l^2}{2M_I M_F}; \quad q^2 = M_I^2 + M_F^2 - 2M_I M_F w. \tag{52}$$

For the $\frac{d\Gamma}{dw}$ we have

$$\frac{d\Gamma}{dw} = \frac{d\Gamma_L}{dw} + \frac{d\Gamma_T}{dw}, \tag{53}$$

where

$$\frac{d\Gamma_L}{dw}(\Xi_{QQ'}^{\diamond} \to \Xi_{QQ'}^{\diamond\prime}) = \frac{G_F^2}{(2\pi)^3}|CKM|^2 \frac{q^2 M_F^2\sqrt{w^2-1}}{12M_I}\{|H_{1/2,0}|^2 + |H_{-1/2,0}|^2\}, \tag{54}$$

$$\frac{d\Gamma_T}{dw}(\Xi_{QQ'}^{\diamond} \to \Xi_{QQ'}^{\diamond\prime}) = \frac{G_F^2}{(2\pi)^3}|CKM|^2 \frac{q^2 M_F^2\sqrt{w^2-1}}{12M_I}\{|H_{1/2,1}|^2 + |H_{-1/2,-1}|^2\}. \tag{55}$$

Here $H_{\lambda_F,\lambda_W} = H^V_{\lambda_F,\lambda_W} - H^A_{\lambda_F,\lambda_W}$, where λ_F and λ_W are helicities of final state baryon and W - boson correspondingly and the functions $H^{V(A)}_{\lambda_F,\lambda_W}$ obey the following symmetry relations:

$$H^{V(A)}_{-\lambda_F,-\lambda_W} = +(-)H^{V(A)}_{\lambda_F,\lambda_W}. \tag{56}$$

The functions remained after the application of this relation can be further expressed in terms of calculated in previous subsection IW - functions with the help of the following formulae

$$H^{V,A}_{1/2,1} = -2\sqrt{M_I M_F(w \mp 1)}\xi^{IW}(w) \tag{57}$$

$$H^{V,A}_{1/2,0} = \frac{1}{\sqrt{q^2}}\sqrt{2M_I M_F(w \mp 1)}(M_I \pm M_F)\xi^{IW}(w) \tag{58}$$

The results of numerical estimates, done with the help of presented formulae can be found in Table 2.

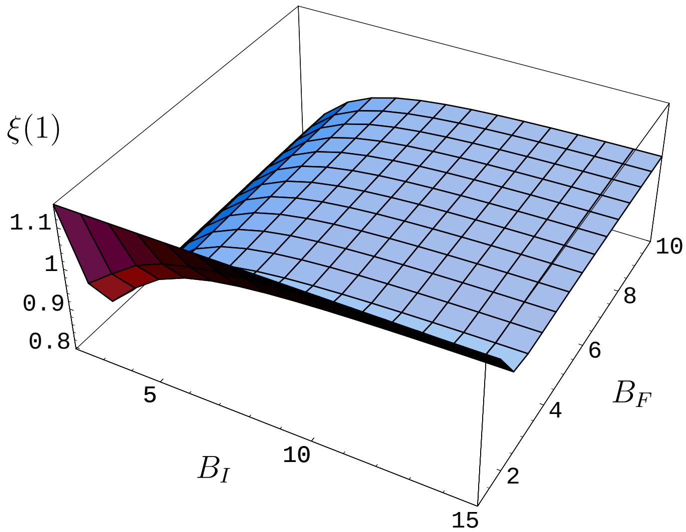


Figure 11: The value of $\xi(1)$ for the transition $\Xi_{cc}^{\diamond} \to \Xi_{c}^{\diamond}$ as function of Borel parameters in the s_I and s_F channels.

To calculate the π or ρ - meson decays we assume the hypothesis of factorization [16]. The corresponding formulae for the decays of doubly heavy baryons with a pion or ρ - meson in the final state can be easily obtained from those for the semileptonic decays by a simple substitution of leptonic tensor by the π - meson current tensor $f_\pi^2 p_\mu^\pi p_\nu^\pi$ or ρ - meson current tensor $f_\rho^2 m_\rho^2(-g_{\mu\nu} + p_\mu p_\nu/m_\rho^2)$

$$\Gamma_{H_I\to H_F\pi} = 6\pi^2 f_\pi^2 a_1^2(\mu)\frac{((M_I+M_F)^2-q^2)}{(M_I+M_F)^2}\frac{d\Gamma}{dq^2}|_{q^2=m_\pi^2}, \quad (59)$$

$$\begin{aligned}\Gamma_{H_I\to H_F\rho} &= \frac{6\pi^2 a_1^2(\mu) f_\rho^2}{(M_I^2-M_F^2)^2}\{(M_I-M_F)^2((M_I+M_F)^2-q^2) + \\ &\quad 2m_\rho^2((M_I-M_F)^2-q^2)\}\frac{d\Gamma}{dq^2}|_{q^2=m_\rho^2}, \end{aligned} \quad (60)$$

where $a_1(\mu) = \frac{1}{2N_c}(C_+(\mu)(N_c+1) + C_-(\mu)(N_c-1))$ and $N_c = 3$ is the number of colors. In numerical calculations we put $a_1 = 1.26$. The results for these nonleptonic transitions can be also found in Table 2. To calculate the branching ratios for exclusive decay modes we used the values of doubly heavy baryon lifetimes, calculated by us previously [5]. There is some difference in concrete numerical values of lifetimes, obtained in different papers [4, 5]. In [5] we have commented on the uncertainties in the resulting values of lifetimes related to the heavy quark mass values. There is, however, one more uncertainty remained, connected with the value of light quark - diquark wave-function at origin. In present there are two approaches to estimate this value: 1) assuming, that this value is the same as the value of D-meson wave-function at origin; 2) extracting this value from the comparison of hyper-fine splittings in doubly and singly heavy baryons. Here we used the estimates for the lifetimes made in the second approach, as they are the most complete ones. The values, presented in Table 2 already include the contribution of spin 1/2-spin 3/2 decay channels. To estimate the latter we have used the results of [11], where the contribution of these channels was calculated for the case of $\Xi_{bc} \to \Xi_{cc} + l\bar{\nu}$ baryon transition, and assumed, that, according to superflavor symmetry, it constitutes 30 % from the contribution of corresponding spin

1/2-spin 1/2 transitions for all transitions between doubly heavy baryons. In calculations of $\Xi_{bb}^{\diamond}$ and $\Xi_{cc}^{\diamond}$ - baryon decay modes we have taken into account a factor 2 due to Pauli principle for the identical heavy quarks in the initial channel. In the case of $\Xi_{bc}^{\diamond} \to \Xi_{cc}^{\diamond\prime} X$-baryon transition the same factor comes from the positive Pauli interference of the c-quark, being a product of b-quark decay, with the c-quark from the initial baryon. Here, we also would like to mention, that for the Ξ_{bc}-baryon decays the mentioned positive Pauli interference contribution is dominant among the other nonspectator contributions[6], so we do not introduce other corrections here. However, in the case of $\Xi_{cc}^{++} \to \Xi_{cs}^{+} X$- baryon transition the negative Pauli interference plays the dominant role and thus should be accounted for explicitly. From the previously done OPE analysis for doubly heavy baryon lifetimes [4, 5] we conclude that the corresponding correction factor in this case is 0.62. We would like also give a small comment on our notations. The $\Xi_{Qs}^{\diamond}$ in Table 2 stays for the sum of $\Xi_{Q}^{\diamond}$ and $\Xi_{Q}^{\diamond\prime}$ decay channels.

Mode	Br (%)	Mode	Br (%)
$\Xi_{bb}^{\diamond} \to \Xi_{bc}^{\diamond} l\bar{\nu}_l$	14.9	$\Xi_{bc}^{+} \to \Xi_{cc}^{++} l\bar{\nu}_l$	4.9
$\Xi_{bc}^{0} \to \Xi_{cc}^{+} l\bar{\nu}_l$	4.6	$\Xi_{bc}^{+} \to \Xi_{bs}^{0} l\nu_l$	4.4
$\Xi_{bc}^{0} \to \Xi_{bs}^{-} l\nu_l$	4.1	$\Xi_{cc}^{++} \to \Xi_{cs}^{+} l\nu_l$	16.8
$\Xi_{cc}^{+} \to \Xi_{cs}^{0} l\nu_l$	7.5	$\Xi_{bb}^{\diamond} \to \Xi_{bc}^{\diamond} \pi^-$	2.2
$\Xi_{bb}^{\diamond} \to \Xi_{bc}^{\diamond} \rho^-$	5.7	$\Xi_{bc}^{+} \to \Xi_{cc}^{++} \pi^-$	0.7
$\Xi_{bc}^{0} \to \Xi_{cc}^{+} \pi^-$	0.7	$\Xi_{bc}^{+} \to \Xi_{cc}^{++} \rho^-$	1.9
$\Xi_{bc}^{0} \to \Xi_{cc}^{+} \rho^-$	1.7	$\Xi_{bc}^{+} \to \Xi_{bs}^{0} \pi^+$	7.7
$\Xi_{bc}^{0} \to \Xi_{bs}^{-} \pi^+$	7.1	$\Xi_{bc}^{+} \to \Xi_{bs}^{0} \rho^+$	21.7
$\Xi_{bc}^{0} \to \Xi_{bs}^{-} \rho^+$	20.1	$\Xi_{cc}^{++} \to \Xi_{cs}^{+} \pi^+$	15.7
$\Xi_{cc}^{+} \to \Xi_{cs}^{0} \pi^+$	11.2	$\Xi_{cc}^{++} \to \Xi_{cs}^{+} \rho^+$	46.8
$\Xi_{cc}^{+} \to \Xi_{cs}^{0} \rho^+$	33.6		

Table 2: Branching ratios for the different decay modes of doubly heavy baryons.

The previous studies of exclusive decays of doubly heavy baryons [11, 12] exploited the spin-flavor symmetry of QCD [17], arising at the very large quark mass limit. The fundamental representation of the SU(6)⊗U(1) spin-flavor symmetry group consists of two-component heavy quark spinor, scalar and axial-vector di-antiquark fields. This representation can be explicitly written in terms of nine-component vector with the four-velocity v:

$$\Psi_v = \begin{pmatrix} h_v \\ S_v \\ A_v^{\mu} \end{pmatrix}$$

where A_v^{μ} satisfies the constraint $v_{\mu} A_v^{\mu} = 0$ and the effective lagrangian for this field is

$$\mathcal{L}_{eff} = \frac{1}{2} \overline{\Psi}_v \mathcal{M}\ iv \cdot \overset{<}{\overline{D}}{}^{>}\ \Psi_v$$

[6]Here we use the results of OPE analysis for the inclusive decay modes of doubly heavy baryons [4, 5]

where $\mathcal{M}$ is a 9×9 mass matrix[7], given by the following expression:

$$\mathcal{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2m_S & 0 \\ 0 & 0 & -2m_A \end{pmatrix}$$

and

$$\overline{\Psi}_v = \Psi_v^\dagger \begin{pmatrix} \gamma^0 \; 0 \; 0 \\ 0 \; 1 \; 0 \\ 0 \; 0 \; g \end{pmatrix}$$

Here g = diag(1,-1,-1,-1) is the usual metric tensor. Next, to make connection with the hadronic states, one considers a tensor product of Ψ_v with one light antiquark field. Thus, this hadronic supermultiplet puts together singly heavy mesons and doubly heavy antibaryons. However, such supermultiplet is not completely flavor-independent even in the heavy quark mass limit, as there remains internal mass-dependent heavy di-antiquark dynamics. The singly heavy baryons with the strangeness in the discussed approach belong to the different supermultiplet and this fact should be taken into account, when calculating form-factors for the semileptonic transitions between doubly heavy and singly heavy baryons. Such analysis within the framework of potential models was performed previously by M.A.Sanchis-Lozano [11] for the case of $\Xi_{bc}^\diamond$-baryon decays. There, to calculate the form-factors, it was assumed that the latter are given by the overlap of Coulomb diquark wave-functions with small non-perturbative corrections, given by the presence of a light quark in the baryons under consideration. It is just the approach we have used in our PM estimates. To reduce the number of independent form-factors, there was performed an analysis of spin-symmetry relations between various form-factors in the limit of zero recoil. The author, using different arguments, had came to the same conclusion as we have did in the present work. That is, all semileptonic transitions of doubly heavy baryons are governed by the only universal function, an analogue of Isgur-Wise function. The numerical results on the normalization of Isgur-Wise function at zero recoil completely agree with our estimates both in the framework of potential models and NRQCD sum rules. The given predictions for the semileptonic decay modes of $\Xi_{bb}^\diamond$-baryons nicely agree with the ones presented in this paper, taking into account correction factor due to the wrong values of $\Xi_{bc}^\diamond$-baryon lifetime used in that paper. There is also a paper, where the diquark semileptonic transitions where calculated within the Bethe-Salpeter approach. However, the numerical results presented in this paper are very strange. It is suffice to say, that, for example, according to these results the semileptonic branching ratios of $\Xi_{bb}^\diamond$-baryon decays should be approximately 50 %, what is very unlikely.

To finish the discussion of the obtained results we would like to note, that the latter are also in agreement with the estimates of inclusive decay channels performed by us previously [4, 5].

5 Conclusion

In this paper we have presented the analysis of exclusive decays of doubly heavy baryons in the framework of NRQCD sum rules. We have provided complete numerical study of baryonic couplings and semileptonic form-factors. The values of semileptonic and some nonleptonic exclusive modes

[7]Note, that the particular form of the mass matrix depends on the fields normalizations.

are also given. To conclude, we would like to discuss what also can be and should be done in the study of exclusive decays of doubly heavy baryons. First, it will be instructive to perform the similar analysis for the baryon currents of the first type. We have check, that these two schemes of calculation give the similar results only in the case of $\Xi_{bb}^{\diamond}$-baryon decays. Second, one may perform an analysis of doubly heavy baryon exclusive decays in full QCD and not relay on the pole resonance model for the form-factors. We plan to present the results of such analysis in nearest future. And, third one may try to calculate the lifetimes of doubly heavy baryons in the framework of QCD sum rules. It is a very interesting task, as we will explicitly see the effect of large nonspectator effects, studied previously in the OPE framework, on various exclusive modes.

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References

[1] F. Abe et al., CDF Collaboration, Phys. Rev. Lett. **81**, 2432 (1998), Phys. Rev. **D58**, 112004 (1998).

[2] S.S.Gershtein, V.V.Kiselev, A.K.Likhoded, A.V.Tkabladze, A.V.Berezhnoy, A.I.Onishchenko, Talk given at 4th International Workshop on Progress in Heavy Quark Physics, Rostock, Germany, 20-22 Sept. 1997, IHEP 98-22 [hep-ph/9803433];
S.S.Gershtein, V.V.Kiselev, A.K.Likhoded, A.V.Tkabladze, Phys. Usp. **38**, 1 (1995) [Usp. Fiz. Nauk **165**, 3 (1995)];
S.S.Gershtein et al., Phys. Rev. **D51**, 3613 (1995).

[3] A.V.Berezhnoy, V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko, Phys. Rev. D57 (1998) 4385;
A.V.Berezhnoy, V.V.Kiselev, A.K.Likhoded, Z.Phys. **A356**, 89 (1996), Phys. Atom. Nucl. **59**, 870 (1996) [Yad. Fiz. **59**, 909 (1996)];
S.P.Baranov, Phys. Rev. D 56, 3046 (1997);
V.V.Kiselev, A.K. Likhoded, M.V. Shevlyagin, Phys. Lett. B332, 411 (1994);
A.Falk et al., Phys. Rev. D49, 555 (1994);
V.V.Kiselev, A.E.Kovalsky, preprint hep-ph/9908321.

[4] V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko, Phys. Rev. **D60**, 014007 (1999), Phys.Atom.Nucl. **62** (1999) 1940, Yad.Fiz. **62** (1999) 2095;
V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko, preprint DESY 98-212 (1999) [hep-ph/ 9901224], to appear in Eur.Phys.J. **C** ;
B.Guberina, B.Melic, H.Stefancic, Eur. Phys. J. **C9**, 213 (1999).

[5] A.I.Onishchenko, preprint hep-ph/9912424;
A.K.Likhoded, A.I.Onishchenko, preprint hep-ph/9912425.

[6] S.S.Gershtein, V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko, preprint IHEP 98-66 (1998) [hep-ph/9811212], Heavy Ion Phys **9**, 133 (1999); [hep-ph/9807375] Mod. Phys. Lett. **A14**, 135 (1999);
D.Ebert, R.N.Faustov, V.O.Galkin, A.P.Martynenko, V.A.Saleev, Z. Phys. C76 (1997) 111;
J.G.K*ö*rner, M.Kr*ä*mer, D.Pirjol, Prog. Part. Nucl. Phys. 33 (1994) 787;
R.Roncaglia, D.B.Lichtenberg, E.Predazzi, Phys. Rev. D52 (1995) 1722.

[7] M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, Nucl. Phys. **B147**, 385 (1979);
L.J.Reinders, H.R.Rubinstein, S.Yazaki, Phys. Rep. **127**, 1 (1985).

[8] E.Bagan, M.Chabab, S.Narison, Phys. Lett. B306 (1993) 350;
E.Bagan at al., Z. Phys. **C64**, 57 (1994).

[9] V.V.Kiselev, A.I.Onishchenko, preprint hep-ph/9909337, to appear in Nucl.Phys. **B**.

[10] V.V.Kiselev, A.E.Kovalsky, preprint hep-ph/0005019.

[11] M.A.Sanchis-Lozano, Nucl.Phys. **B440** (1995) 251.

[12] X.-H.Guo, H.-Y.Jin, X.-Q.Li, Phys.Rev **D58** (1998), 114007.

[13] A.Smilga, Yad. Phys. **35**, 473 (1982).

[14] R.E.Cutkosky, J. Math. Phys. **1**, 429 (1960).

[15] V.V.Kiselev, A.V.Tkabladze, Phys. Rev. **D48**,(1993), 5208;
V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko, Nucl.Phys. **B569**, (2000), 473; V.V.Kiselev, A.E.Kovalsky, A.K.Likhoded, hep-ph/0002127

[16] M.Dugan and B.Grinstein, Phys.Lett. **B255** (1991) 583;
M.A.Shifman, Nucl.Phys. **B388** (1992) 346;
B.Blok, M.Shifman, Nucl.Phys. **389** (1993) 534.

[17] H.Georgi and M.B.Wise, Phys.Lett. **B243** (1990) 279;
C.D.Carone, Phys.Lett. **B253** (1991) 408;
M.J.Savage and M.B.Wise, Phys.Lett. **B248** (1990) 177.

Appendix A

In this Appendix we have collected theoretical expressions for spectral densities of Wilson coefficients, standing in front of various operators, obtained as the result of OPE expansion of two-point correlation function.

For the case of $\epsilon^{\alpha\beta\lambda}:(Q'^T_\alpha C\gamma_5 q_\beta)Q_\lambda:$ current we have

$$\rho_1^{pert} = \frac{2\sqrt{2}\sqrt{m_1m_2(m_1+m_2)}}{105(m_1+m_2)^3\pi^3}w^{7/2}(m_1m_2(12m_2-13w)+5m_2^2w+m_1^2(12m_2+5w)) \tag{61}$$

$$\rho_2^{pert} = \frac{2\sqrt{2}\sqrt{m_1m_2(m_1+m_2)}}{105(m_1+m_2)^2\pi^3}w^{7/2}(m_1m_2(12m_2-w)+m_2^2w+m_1^2(12m_2+5w)) \tag{62}$$

$$\rho_1^{\bar{q}q} = -\frac{\sqrt{m_1m_2(m_1+m_2)}}{4\sqrt{2}(m_1+m_2)^3\pi}\sqrt{w}(m_1m_2(4m_2-5w)+5m_2^2w+m_1^2(4m_2+w)) \tag{63}$$

$$\rho_2^{\bar{q}q} = -\frac{\sqrt{m_1m_2(m_1+m_2)}}{4\sqrt{2}(m_1+m_2)^2\pi}\sqrt{w}(m_1m_2(4m_2-w)+m_2^2w+m_1^2(4m_2+w)) \tag{64}$$

$$\begin{aligned}\rho_1^{G^2} = {} & \frac{1}{1536\sqrt{2}\pi(m_1m_2)^{3/2}(m_1+m_2)^{7/2}}\sqrt{w}(2m_2^5w^2-28m_1^4m_2w(4m_2+w)+ \\ & m_1m_2^4w(-16m_2+35w)+m_1^5(32m_2^2+8m_2w-w^2)- \\ & 8m_1^2m_2^3(8m_2^2-13m_2w+28w^2)+m_1^3(-96m_2^4+217m_2^2w^2))\end{aligned} \tag{65}$$

$$\begin{aligned}\rho_2^{G^2} = {} & \frac{1}{7680\sqrt{2}\pi(m_1m_2)^{3/2}(m_1+m_2)^{5/2}}\sqrt{w}(122m_2^5w^2-100m_1^4m_2(4m_2+w)+ \\ & m_1m_2^4w(880m_2+327w)-40m_1^2m_2^3(8m_2^2-41m_2w-20w^2)+ \\ & 5m_1^2(32m_2^2+8m_2w-w^2)+5m_1^3m_2^2(-96m_2^2+64m_2w+169w^2))\end{aligned} \tag{66}$$

$$\begin{aligned}\rho_1^{mix} = {} & \frac{1}{2048\sqrt{2}\pi(m_1m_2)^{1/2}(m_1+m_2)^{7/2}\sqrt{w}}(m_1^2m_2^2(64m_2-397w)+ \\ & 30m_1m_2^3(4m_2-27w)+105m_2^4w+m_1^4(-104m_2+17w)+10m_1^3m_2(-16m_2+19w))\end{aligned} \tag{67}$$

$$\begin{aligned}\rho_2^{mix} = {} & \frac{1}{2048\sqrt{2}\pi(m_1m_2)^{1/2}(m_1+m_2)^{5/2}\sqrt{w}}(2m_1^3m_2(96m_2-43w)- \\ & 6m_1m_2^3(4m_2-23w)+15m_2^4w+m_1^4(104m_2-17w)+m_1^2m_2^2(64m_2+45w))\end{aligned} \tag{68}$$

Here m_1 is the mass of Q-quark and m_2 is the mass of Q'-quark. For the case of $\epsilon^{\alpha\beta\lambda}:(Q^T_\alpha C\gamma_5 q_\beta)s_\lambda:$ current we have

$$\rho_1^{pert} = \frac{w^3}{80m_Q^2\pi^3}(135m_s^2w^2-12m_Qm_sw(5m_s+3w)+4m_Q^2(5m_s^2+5m_sw+w^2)) \tag{69}$$

$$\rho_2^{pert} = \frac{m_sw^3}{40m_Q\pi^3}(9m_sw^2+5m_Q^2(4m_s+w)-m_Qw(5m_s+w)) \tag{70}$$

$$\begin{aligned}\rho_1^{\bar{q}q} = {} & -\frac{1}{4m_Q^3\pi}(-85m_s^2w^3-m_Q^2w(3m_s+2w)^2+m_Qm_sw^2(33m_s+26w)+ \\ & m_Q^3(ms^2+4m_sw+2w^2))\end{aligned} \tag{71}$$

$$
\begin{aligned}
\rho_2^{\bar{q}q} &= -\frac{m_Q m_s}{4\pi(m_Q+w)^2}(2m_Q^2(m_s+w)+w^2(m_s+w)+m_Q w(2m_s+3w)) && (72)\\
\rho_1^{G^2} &= \frac{1}{1536\pi m_Q^4}(-2185m_s^2w^3-28m_Q^2w(3m_s+2w)^2+2m_Qm_sw^2(437m_s+344w)- \\
&\quad 32m_Q^4(m_s+w)+32m_Q^3(m_s^2+4m_sw+2w^2)) && (73)\\
\rho_2^{G^2} &= -\frac{1}{384\pi m_Q^2}(m_Qm_sw(6m_s-13w)+4m_s^2w^2+8m_Q^2m_s(w(\log 8-4)+m_s(\log 8-1))- \\
&\quad 8m_Q^3(m_s+4w-m_s\log 8)+24m_Q^2m_s(m_Q+m_s+w)\log w) && (74)\\
\rho_1^{\bar{q}Gq} &= \frac{1}{16m_Q^4\pi}(m_Q^4+168m_s^2w^2-5m_Q^3(m_s+w)-3m_Qm_sw(18m_s+19w)+ \\
&\quad m_Q^2(10m_s^2+22m_sw+11w^2)) && (75)\\
\rho_2^{\bar{q}Gq} &= \frac{m_s}{16m_Q^3\pi}(-m_Q^3+3m_sw^2+m_Q^2(m_s+w)-m_Qw(2m_s+w)) && (76)\\
\rho_2^{\bar{s}Gs} &= \frac{m_Q+m_s+w}{16\pi} && (77)
\end{aligned}
$$